Djairo G. de Figueiredo • Jean-Pierre Gossez • Pedro Ubilla

Multiplicity results for a family of semilinear elliptic problems under local superlinearity and sublinearity

Received June 12, 2005 and in revised form November 7, 2005

Abstract. We study the existence, nonexistence and multiplicity of positive solutions for the family of problems $-\Delta u = \lambda \phi(u) + \psi(u)$, $u \in W_0^1,1(\Omega)$, where $\Omega$ is a bounded domain in $\mathbb{R}^N$, $N \geq 3$ and $\lambda > 0$ is a parameter. The results include the well-known nonlinearities of the Ambrosetti–Prodi–Cerami type in a more general form, namely $\lambda \phi(u) = a(u)u^q + b(u)u^p$, where $0 < q < 1 < p \leq \frac{2N}{N-2} - 1$. The coefficient $a(u)$ is assumed to be nonnegative but $b(u)$ is allowed to change sign, even in the critical case. The notions of local superlinearity and local sublinearity introduced in [9] are essential in this more general framework. The techniques used in the proof are lower and upper solutions and variational methods.

Keywords. Multiplicity, semilinear elliptic problem, local sub- and superlinearity nonlinearities, concave-convex nonlinearities, critical exponent, upper and lower solutions, variational method.