Representations of Finite Groups

Organised by
Joseph Chuang, London
Markus Linckelmann, Aberdeen
Gunter Malle, Kaiserslautern
Jeremy Rickard, Bristol

March 22nd – March 28th, 2009

Abstract. The workshop Representations of Finite Groups was organised by Joseph Chuang (London), Markus Linckelmann (Aberdeen), Gunter Malle (Kaiserslautern) and Jeremy Rickard (Bristol). It covered a wide variety of aspects of the representation theory of finite groups and related objects like Hecke algebras. A particular focus was placed on the rapidly evolving area of fusion systems.


Introduction by the Organisers

The workshop Representations of Finite Groups was organised by Joseph Chuang (London), Markus Linckelmann (Aberdeen), Gunter Malle (Kaiserslautern) and Jeremy Rickard (Bristol). It was attended by 46 participants with broad geographic representation. It covered a wide variety of aspects of the representation theory of finite groups and related objects like Hecke algebras. A particular focus was placed on recent developments on fusion systems. This area, which sits somewhere between group theory, block theory and homotopy theory, has received a significant amount of attention since the breakthrough papers of Broto, Levi, Oliver in the last six or seven years.

In twelve longer lectures of 40 minutes each and seventeen shorter contributions of 30 minutes each, recent progress in representation theory was presented and interesting new research directions were proposed. Besides the lectures, there was plenty of time for informal discussion between the participants, either continuing ongoing research cooperation or starting new projects.
Several interesting new results were presented related to homological methods in representation theory.

Symonds sketched his recent proof, using equivariant cohomology, of a five-year old conjecture of Benson on the commutative algebra of the cohomology ring of a general finite group: that its Castelnuovo-Mumford regularity is always zero, a conjecture for which a lot of computational evidence had been accumulating recently.

Benson and Carlson both spoke on topics related to the Jordan type of modules for finite groups, a subject that originated with a finer study of the notion of rank varieties for modules, and which has recently found unexpected connections with other areas of mathematics. In particular, Benson gave a survey of conjectures and results on modules of constant Jordan type, including work of him and Pevtsova relating these to vector bundles on projective spaces.

Xu described his work on the cohomology of categories, including his surprisingly simple construction of a finite-dimensional algebra whose Hochschild cohomology is not finitely generated modulo nilpotent elements. This answers in the negative a question of Snashall, and has implications on extending the theory of cohomological varieties for representations of finite groups to representations of more general algebras.

Navarro and Tiep presented their proof of Brauer’s longstanding height zero conjecture for 2-blocks of maximal defect. Eaton reported on the proof of the fact that every nilpotent block of a finite simple group must have abelian defect groups.

Geck described his construction of natural labels for modular principal series representations of finite groups of Lie-type which might point a way towards a proof of James’ conjecture on decomposition numbers. Bonnafé showed that Lusztig’s conjectures (P1)-(P15) on Hecke algebras with unequal parameters are compatible with the parametrizations of simple modules coming from Ariki’s Theorem.

Amongst the talks on fusion systems, one of the highlights was the characterisation, by Ragnarsson and Stancu, of fusion systems in terms of a reciprocity property in double Burnside rings. This characterisation bypasses the usual axiomatic description of fusion systems, and may well open new territory. One of the fundamental problems in block theory which can be formulated in terms of fusion systems without reference to blocks - the 2-cocycle gluing problem - was shown by Park to have more than one solution in certain cases (it remains an open question whether this problem has always at least one solution). The other fundamental open problem in this area is the question as to whether every fusion system has a centric linking system - and a conjecture of Oliver plays this back to finite $p$-groups. Mazza presented joint work with D. Green and L. Hethelyi proving special cases of Oliver’s conjecture, implying in particular the existence of centric linking systems in those cases.

Bob Oliver reported on joint work with C. Broto and J. Møller, in which homotopy theoretic methods are used in order to obtain a sufficient criterion for two finite groups of Lie type to have equivalent fusion systems.