

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mini-Workshop: L^2 -Spectral Invariants and the Integrated Density of States

Organised by

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ABSTRACT. L^2 -spectral invariants play an increasingly important role in the analysis of infinite geometric objects allowing for the action of a group. Typical such objects are covering spaces like Riemannian manifolds and graphs. The aim is to understand the group and the geometry of the object. The associated L^2 -invariants can all be derived from the *integrated density of states* —also known as *spectral distribution function*— of a suitable geometrically induced equivariant Laplacian. On the other hand, the integrated density of states is also a most prominent quantity in the study of Laplacians with additional (dis)order included. Such operators arise in Mathematical Physics and are known as random Schrödinger operators or more general equivariant Hamiltonians. Here, the aim is to understand spectral consequences of the underlying (dis)order.

While overall aims and specific perspectives on the integrated density of states may be somewhat different, it turns out that typical questions in both contexts concern its computation by averaging procedures, its continuity features at certain points of the spectrum and some logarithmic integrals.

Mathematics Subject Classification (2000): IMU-classifications : 13, 4, 9.

Introduction by the Organisers

Both the study of L^2 -spectral invariants in geometry and the investigation of the integrated density of states in mathematical physics have attracted much attention in recent years. While the two topics are strongly related, the corresponding communities are rather unaware of each others work and methods. The main aim

of this mini-workshop was to bring together people from both fields and provide a basis for interaction.

Accordingly, the first two days of the conference were spent with survey talks solicited by the organizers to highlight concepts and methods. There were 9 such talks with durations between 60 and 90 minutes. The second half of the conference was devoted to more detailed investigations. Most participants used the opportunity to present their current research in the area of the meeting. There were 13 such talks.

The results presented in those talks contained significant contributions e.g. to the Atiyah conjecture about integrality of L^2 -Betti numbers for a completely new class of groups by Peter Linnell, a mathematically rigorous derivation using von Neumann traces of the asymptotics of the specific heat near absolute zero by Mikhael Shubin, and approximation results for the integrated density of states in various new contexts.

Altogether the conference was attended by 17 participants.