

Contents

| | |
|--|-----|
| Prologue | 1 |
| 1 Fluid mechanics and the shock development problem | 43 |
| 1.1 General equations of motion | 43 |
| 1.2 The irrotational case and the nonlinear wave equation | 54 |
| 1.3 The non-relativistic limit | 58 |
| 1.4 Jump conditions | 73 |
| 1.5 The shock development problem | 87 |
| 1.6 The restricted shock development problem | 97 |
| 2 Geometric construction | 101 |
| 2.1 General construction in Lorentzian geometry | 101 |
| 2.2 The characteristic system | 107 |
| 2.3 The wave system | 114 |
| 2.4 Variations by translations and the wave equation for the rectangular components of β | 116 |
| 2.5 Geometric construction for the shock development problem | 125 |
| 3 Acoustical structure equations | 129 |
| 3.1 Connection coefficients and the first variation equations | 129 |
| 3.2 Structure functions and the formulas for the torsion forms | 135 |
| 3.3 Propagation equations for λ and $\underline{\lambda}$ | 147 |
| 3.4 Second variation and cross variation equations | 148 |
| 3.5 The case $n = 2$ | 153 |
| 3.6 The Codazzi and Gauss equations ($n > 2$) | 158 |
| 4 The problem of the free boundary | 165 |
| 4.1 Analysis of the boundary conditions | 165 |
| 4.2 Transformation functions and identification equations | 171 |
| 4.3 Regularization of identification equations | 193 |
| 5 Initial data and derived data | 199 |
| 5.1 Propagation equations for $\underline{\lambda}$ and s_{NL} on $\underline{\mathcal{C}}$ | 199 |
| 5.2 Propagation equations for higher-order derived data $T^m \underline{\lambda}$ and $T^m s_{NL}$ on $\underline{\mathcal{C}}$ | 208 |
| 5.3 Boundary conditions for higher-order derived data and determina- tion of the T -derivatives of the transformation functions on $\partial \underline{\mathcal{B}}$ | 211 |

| | | |
|------|---|-----|
| 6 | Variation fields | 229 |
| 6.1 | Bi-variational stress | 229 |
| 6.2 | Variation fields V and associated 1-forms ${}^{(V)}\theta^\mu$ | 233 |
| 6.3 | Fundamental energy identities | 239 |
| 6.4 | Boundary condition on \mathcal{K} for the 1-forms ${}^{(V)}\xi$ | 246 |
| 7 | Multiplier field | 267 |
| 7.1 | Coercivity at the boundary. Choice of multiplier field | 267 |
| 7.2 | Deformation tensor of the multiplier field. Error integral associated to ${}^{(V)}Q_1$ | 277 |
| 7.3 | Error integral associated to ${}^{(V)}Q_2$ | 286 |
| 8 | Commutation fields | 297 |
| 8.1 | Commutation fields and higher-order variations | 297 |
| 8.2 | Recursion formulas for source functions | 298 |
| 8.3 | Deformation tensors of the commutation fields | 305 |
| 8.4 | Principal acoustical error terms | 309 |
| 9 | Power series approximation method | 319 |
| 9.1 | Setup of the truncated power series | 319 |
| 9.2 | Estimates for the quantities by which the N th approximants fail to satisfy the characteristic and wave systems | 322 |
| 9.3 | Estimates for the quantities by which the N th approximants fail to satisfy the boundary conditions | 337 |
| 9.4 | Estimates for the quantities by which the N th approximants fail to satisfy the identification equations | 345 |
| 9.5 | Estimates for the quantity by which the $\beta_{\mu,N}$ fail to satisfy the wave equation relative to \tilde{h}_N and to \tilde{h}'_N | 348 |
| 9.6 | Variation differences ${}^{(m,l)}\check{\phi}_\mu$ and rescaled source differences ${}^{(m,l)}\check{\rho}_\mu$ | 362 |
| 9.7 | Difference 1-forms ${}^{(V;m,l)}\check{\xi}$. Difference energies and difference energy identities | 368 |
| 10 | Top-order acoustical estimates in the case $d = 2$ | 373 |
| 10.1 | Regularization of the propagation equations for $\check{\chi}$ and $\check{\underline{\chi}}$ | 373 |
| 10.2 | Regularization of the propagation equations for $E^2\lambda$ and $E^2\underline{\lambda}$ | 379 |
| 10.3 | Structure equations for the N th approximants | 390 |
| 10.4 | Propagation equations for $\check{\theta}_l, \check{\underline{\theta}}_l$ and for $\check{v}_{m,l}, \check{\underline{v}}_{m,l}$ | 408 |
| 10.5 | Estimates for $\check{\theta}_l$ | 426 |
| 10.6 | Estimates for $\check{v}_{m-1,l+1}$ | 448 |
| 10.7 | Estimates for $\check{\underline{\theta}}_l$ and $\check{\underline{v}}_{m-1,l+1}$ in terms of their boundary values on \mathcal{K} | 471 |

| | | |
|-------|--|-----|
| 10.8 | Boundary conditions on \mathcal{K} and preliminary estimates for $\check{\theta}_l$ and $\check{\gamma}_{m-1,l+1}$ on \mathcal{K} | 498 |
| 11 | Outline of top-order acoustical estimates for more than 2 spatial dimensions | 513 |
| 12 | Top-order estimates for transformation functions and next-to-top-order acoustical estimates | 543 |
| 12.1 | Propagation equations for the next-to-top-order acoustical difference quantities $(^{(n-1)}\check{\chi}, ^{(n-1)}\check{\underline{\chi}})$ and $(^{(m,n-m)}\check{\lambda}, ^{(m,n-m)}\check{\underline{\lambda}}) : m = 0, \dots, n$ | 543 |
| 12.2 | Estimates for $(^{(n-1)}\check{\chi}, ^{(n-1)}\check{\underline{\chi}})$ and $(^{(0,n)}\check{\lambda}, ^{(0,n)}\check{\underline{\lambda}})$ | 566 |
| 12.3 | Estimates for $(T\Omega^n \check{f}, T\Omega^n \check{v}, T\Omega^n \check{\gamma})$ | 635 |
| 12.4 | Estimates for $(^{(m,n-m)}\check{\lambda}, ^{(m,n-m)}\check{\underline{\lambda}}) : m = 1, \dots, n$ | 649 |
| 12.5 | Estimates for $(T^{m+1}\Omega^{n-m}\check{f}, T^{m+1}\Omega^{n-m}\check{v}, T^{m+1}\Omega^{n-m}\check{\gamma}) : m = 1, \dots, n$ | 706 |
| 12.6 | Estimates for $(\Omega^{n+1}\check{f}, \Omega^{n+1}\check{v}, \Omega^{n+1}\check{\gamma})$ | 720 |
| 13 | Top-order energy estimates | 741 |
| 13.1 | Estimates for $^{(V;m,n-m)}\check{b}$ | 741 |
| 13.2 | Borderline error integrals contributed by $^{(V;m,n-m)}\check{Q}_1, ^{(V;m,n-m)}\check{Q}_2$ | 754 |
| 13.3 | Borderline error integrals associated to $\check{\theta}_n$ and to $\check{\gamma}_{m-1,n-m+1} : m = 1, \dots, n$ | 755 |
| 13.4 | Borderline error integrals associated to $\check{\theta}_n$ and to $\check{\gamma}_{m-1,n-m+1} : m = 1, \dots, n$ | 765 |
| 13.5 | Top-order energy estimates | 777 |
| 14 | Lower-order estimates, recovery of the bootstrap assumptions, and completion of the argument | 791 |
| 14.1 | Summary of the preceding, outline of the following, and statement of the theorem | 791 |
| 14.2 | Bootstrap assumptions needed | 797 |
| 14.3 | $L^2(S_{\underline{u},u})$ estimates for $^{(n-1)}\check{\chi}$ and for $^{(m,n-m)}\check{\lambda} : m = 0, \dots, n - 1$. | 806 |
| 14.4 | $L^2(\mathcal{K}_\sigma^\tau)$ estimates for the n th-order acoustical differences | 819 |
| 14.5 | $L^2(S_{\underline{u},u})$ estimates for the n th-order variation differences | 828 |
| 14.6 | $L^2(S_{\underline{u},u})$ estimates for the $(n-1)$ th-order acoustical differences . . . | 842 |
| 14.7 | $L^2(S_{\underline{u},u})$ estimates for all n th-order derivatives of the β_μ | 851 |
| 14.8 | $L^2(S_{\underline{u},u})$ estimates for $\Omega^{n-1} \log \#$ and $\Omega^{n-1} b$ | 855 |
| 14.9 | Lower-order $L^2(S_{\underline{u},u})$ estimates | 860 |
| 14.10 | Pointwise estimates and recovery of the bootstrap assumptions . . . | 863 |
| 14.11 | Completion of the argument | 869 |
| | Bibliography | 915 |
| | Index | 917 |