

Contents

Introduction	1
1 Behavior near the boundary of solutions to the Dirichlet problem for a second-order elliptic equation	7
1.1 Capacitary modulus of continuity of a harmonic function	7
1.2 Operator in divergence form with measurable bounded coefficients	11
1.2.1 Notation and lemmas	11
1.2.2 Estimates of the solution with finite energy integral	14
1.2.3 Estimates for solutions with unbounded Dirichlet integral and the Phragmén–Lindelöf principle	17
1.2.4 Nonhomogeneous boundary condition	18
1.2.5 Nonhomogeneous equation	20
1.3 Refined estimate for the modulus of continuity of a harmonic function	21
1.4 Improvement of previous estimates for \mathcal{L} -harmonic functions	32
1.5 More notations and preliminaries	34
1.6 \mathcal{L} -harmonic functions vanishing on a part of the boundary	36
1.7 Behaviour of \mathcal{L} -harmonic functions at infinity and near a singular point	46
1.8 Phragmén–Lindelöf type theorems	49
1.9 \mathcal{L} -harmonic measure and non-homogeneous Dirichlet data	51
1.10 The Green function and solutions of the non-homogeneous equation	56
1.11 Continuity modulus of solutions and criterion of Hölder regularity of a point	60
1.12 Sufficient conditions for Hölder regularity	63
1.13 Comments to Chapter 1	65
2 An analogue of the Wiener criterion for the Zaremba problem for the Laplacian in a half-cylinder	67
2.1 Formulation of the Zaremba problem	67
2.2 Auxiliary assertions	69
2.3 Estimates for solutions of the Zaremba problem	71
2.4 Regularity criterion for the point at infinity	74
2.5 Estimates for the Green function and for the harmonic measure of the Zaremba problem	81
2.6 Comments to Chapter 2	85
3 Wiener type test for the Zaremba problem for degenerate elliptic operators in a half-cylinder	87
3.1 Introduction	87
3.2 Weighted function spaces and weak solutions	89

3.3	Change of variables	93
3.4	Regularity test	97
3.5	The capacity cap_Γ	102
3.6	The capacity $\text{cap}_\mathcal{K}$	106
3.7	Comments to Chapter 3	112
4	Modulus of continuity of solutions to quasilinear elliptic equations . . .	113
4.1	Preliminaries	113
4.2	Main result	125
4.3	Comments to Chapter 4	130
5	Discontinuous solution to the p-Laplace equation	133
5.1	Construction of a special solution	133
5.2	Asymptotic formula for the Hölder exponent	140
5.3	Behavior of solutions to the equation $\Delta_p u = 0$	147
5.3.1	Absence of Hölder continuity	147
5.3.2	Absence of continuity	148
5.4	Comments to Chapter 5	151
6	Wiener test for higher-order elliptic equations	153
6.1	Introduction	153
6.2	Capacities and the L -capacitary potential	155
6.3	Weighted positivity of $L(\partial)$	163
6.4	Further properties of the L -capacitary potential	166
6.5	Poincaré inequality with m -harmonic capacity	167
6.6	Proof of sufficiency in Theorem 6.1.2	169
6.7	Equivalence of two definitions of regularity	172
6.8	Regularity as a local property	173
6.9	Proof of necessity in Theorem 6.1.2	174
6.10	Proof of sufficiency in Theorem 6.1.1	176
6.11	Proof of necessity in Theorem 6.1.1	179
6.12	The biharmonic equation in a domain with inner cusp ($n \geq 8$)	187
6.13	Comments to Chapter 6	190
7	Wiener test for the polyharmonic equation	191
7.1	Weighted positivity of $(-\Delta)^m$	191
7.2	Local estimates	199
7.3	Pointwise estimates for the Green function	201
7.4	Comments to Chapter 7	203
8	Weighted positivity and Wiener regularity of a boundary point for the fractional Laplacian	205
8.1	Introduction	205
8.2	Notations and preliminaries	206
8.3	Weighted positivity of $(-\Delta)^\mu$	208

8.4	Proof of Lemma 8.3.2	209
8.5	Non-positivity	214
8.6	Local estimates	218
8.7	Regularity of a boundary point	224
8.8	Comments to Chapter 8	226
9	Wiener type regularity of a boundary point for the 3D Lamé system	227
9.1	Statement of results	227
9.2	Proof of Theorem 9.1.1	228
9.3	Proof of Theorem 9.1.2	240
9.4	Comments to Chapter 9	245
10	Boundedness of the gradient of a solution and Wiener test of order one for the biharmonic equation	247
10.1	Introduction	247
10.2	Integral identity and global estimate	250
10.3	Local energy and L^2 estimates	253
10.4	Estimates for the Green function	258
10.5	The capacity Cap_p	263
10.6	1-Regularity of a boundary point	268
10.7	Sufficient condition for 1-regularity	270
10.8	Necessary condition for 1-regularity	276
10.9	Examples and further properties of Cap_p and Cap	286
10.10	Comments to Chapter 10	295
11	Boundedness of derivatives of solutions to the Dirichlet problem for the polyharmonic equation	297
11.1	Introduction	297
11.2	Integral inequalities and global estimate: the case of odd dimension. Part I: power weight	300
11.3	Preservation of positivity for solutions of ordinary differential equations	309
11.4	Integral inequalities and global estimate: the case of odd dimension. Part II: weight g	315
11.5	Integral identity and global estimate: the case of even dimension. Part I: power-logarithmic weight	324
11.6	Integral identity and global estimate: the case of even dimension. Part II: weight g	327
11.7	Pointwise and local L^2 estimates for solutions to the polyharmonic equation	338
11.8	Green function estimates	344
11.9	Estimates for solutions of the Dirichlet problem	353
11.10	Comments to Chapter 11	355

12 Polyharmonic capacities and higher-order Wiener test	357
12.1 Introduction	357
12.2 Regularity of solutions to the polyharmonic equation	361
12.3 Higher-order regularity of a boundary point as a local property	368
12.4 The new notion of polyharmonic capacity	373
12.5 Sufficient condition for λ -regularity	383
12.5.1 Poincaré-type inequalities	383
12.5.2 Odd dimensions	390
12.5.3 Even dimensions	396
12.6 Necessary condition for λ -regularity	397
12.6.1 Fine estimates on the quadratic forms	397
12.6.2 Scheme of the proof	402
12.6.3 Main estimates. Bounds for auxiliary functions T and W related to polyharmonic potentials on the spherical shells	403
12.6.4 Conclusion of the proof	416
12.7 Comments to Chapter 12	418
Bibliography	419
General Index	429
Index of Mathematicians	431