

# Preface

The Second International Conference and Workshop on Valuation Theory was held in Segovia and El Escorial, Spain, July 18-29, 2011. It covered several of the recent developments in valuation theory and its applications. Valuation theory developed in the first half of the last century as a part of algebraic number theory, the theory of ordered fields and the theory of ordered abelian groups. Then, through the work of Krull, Zariski and Abhyankar, valuation theory found important applications in algebraic geometry. Later, as the expected applications in algebraic geometry were obtained by other methods, and algebraic number theory did not provide any striking new valuation theoretical questions, research in the topic became less intense. However, mathematicians from real algebra and from model theory kept a keen interest in valuation theory, and there was a remarkable development of the subject, albeit somewhat distant from the main stream of algebraic geometry. In recent years, valuation theory has found its way back into algebraic geometry and many important applications in various branches of mathematics have been discovered. It is fascinating to see how fundamental principles of valuation theory are found to play a role in various topics which seem to be only loosely related.

As important examples, valuation theory has become extremely useful in the theory of complex dynamical systems, and in the study of non-oscillating trajectories of real analytic vector fields in three dimensions. Analogues of the Riemann-Zariski valuation spaces have been found to be the proper framework for questions of intersection theory in algebraic geometry and in the analysis of singularities of complex plurisubharmonic functions.

In a different direction, the relation between Berkovich geometry, tropical geometry and valuation spaces, on the one hand, and the geometry of arc spaces and valuation spaces, on the other, have begun to deepen and clarify.

Ever since its beginnings, valuation theory and Galois theory have grown closely together and influenced each other. Arguably, studying and understanding the extensions of valuations in algebraic field extensions is one of the most important questions in valuation theory, whereas using valuation theory is one of the most important tools in studying Galois extensions of fields, as well as constructing field extensions with given properties.

The well established topic of the model theory of valued fields is also being transformed, in particular through the study of valued fields with functions and operators, and through the study of types over valued fields.

Simultaneously, the work on the “classical” aspects of valuation theory has increased and substantial progress has been made toward the solution of the local uniformization problem in arbitrary characteristic, the study of ramification of valuations in arbitrary dimension, and in particular the nature and role of the mysterious “defect”.

All of this corresponds to new insights and new approaches to old (and often longstanding) problems. This is why right now is a perfect time to hold conferences on the newly growing and flourishing subject of valuation theory.

The organizers of the Second International Conference and Workshop on Valuation Theory chose to privilege the topics listed above among a great diversity of topics where valuation theory plays a role, but some of those other topics were represented as well, and some contributed lectures were accepted. The audience included graduate students and researchers in the various directions of this subject. The conference brought together the experts of several branches of valuation theory and of related topics in order to strengthen the relations between these branches. At the same time it introduced non-experts to the principles and results of valuation

theory, in particular when they were coming from areas of mathematics in which valuation theory has recently witnessed important contributions (e.g., algebraic geometry, Galois theory, asymptotic analysis, dynamical systems) or in which the role of valuation theory still has to be determined (e.g.,  $C^*$ -algebras). One main goal of our conference was to point out and study the relations between different branches of valuation theory, and their applications to other branches of mathematics. Another main goal was to state and discuss the many important open problems in valuation theory in order to provide an optimal basis for future research conducted by experts and students.

The scientific committee of the conference consisted of Bernard Teissier (chair), Antonio Campillo, Zoe Chatzidakis, Dale Cutkosky, Lou van den Dries, Charles Favre, Ehud Hrushovski, Kiran Kedlaya and Francois Loeser. The members of the organizing committee were Franz-Viktor Kuhlmann (chair), Félix Delgado, Antonio Díaz-Cano, Ignacio Luengo, Florian Pop and Michel Vaquié. The local organizing committee included Santiago Encinas (chair), Maria Alberich, Ignacio Farrán, Carlos Galindo, Evelia García Barroso and Pedro González Pérez.

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