

Contents

1	Introduction	5
1.1	Motivation for cluster algebras	5
1.2	Some recurrences	7
1.3	Somos recurrences	8
1.4	Why study cluster algebras?	9
1.5	Some notation	9
2	Cluster algebras	10
2.1	Definition of a cluster algebra	10
2.2	Skew-symmetrizable matrices	13
2.3	Quiver notation	13
2.4	Valued quiver notation	16
2.5	Exchange graphs	17
3	Exchange pattern cluster algebras	19
3.1	Exchange patterns	19
3.2	Exchange pattern cluster algebras	22
3.3	Matrices of exponents	23
4	Reflection groups	31
4.1	Definition of a reflection group	31
4.2	Root systems	33
4.3	Simple systems	35
4.4	Coxeter groups	36
4.5	Crystallographic root systems and Cartan matrices	37
4.6	Classification of finite crystallographic reflection groups	39
4.7	Type A_n	39
4.8	Root systems of low rank	41
4.9	Finite Coxeter groups	43
4.10	Reduced expressions	44
4.11	Coxeter elements	47
5	Cluster algebras of finite type	49
5.1	Classification	49
5.2	Folding	53
5.3	Denominators	55
5.4	Root clusters	58
5.5	Admissible sequences of sinks	59
5.6	Number of clusters	62

6	Generalized Associahedra	63
6.1	Fans	63
6.2	The cluster fan	64
6.3	The cluster complex	67
6.4	Normal fans	69
6.5	The generalized associahedron	70
6.6	The generalized associahedron of type A_n	71
7	Periodicity	75
7.1	The Somos-4 recurrence	75
7.2	Period 1 quivers	76
7.3	Periodicity in the coefficient case	78
7.4	Higher period quivers	80
7.5	Categorical periodicity	80
8	Quivers of finite mutation type	84
8.1	Classification	84
8.2	Tagged triangulations	87
9	Grassmannians	90
9.1	Exterior powers	90
9.2	The Grassmannian	91
9.3	The Grassmannian $Gr(2, n)$	92
9.4	The Grassmannian $Gr(k, n)$	94
9.5	The Grassmannian $Gr(2, n)$ revisited	98
	Bibliography	101
	Nomenclature	111
	Index	115