Foreword

These lecture notes describe a way of looking at extremes in a multivariate setting. We shall introduce a continuous one-parameter family of multivariate generalized Pareto distributions that describe the asymptotic behaviour of exceedances over linear thresholds. The one-dimensional theory has proved to be important in insurance, finance and risk management. It has also been applied in quality control and meteorology. The multivariate limit theory presented here is developed with similar applications in mind. Apart from looking at the asymptotics of the conditional distributions given the exceedance over a linear threshold – the so-called high risk scenarios – one may look at the behaviour of the sample cloud in the given direction. The theory then presents a geometric description of the multivariate extremes in terms of limiting Poisson point processes.

Our terminology distinguishes between extreme value theory and the limit theory for coordinatewise maxima. Not all extreme values are coordinatewise extremes! In the univariate theory there is a simple relation between the asymptotics of extremes and of exceedances. One of the aims of this book is to elucidate the relation between maxima and exceedances in the multivariate setting. Both exceedances over linear and elliptic thresholds will be treated. A complete classification of the limit laws is given, and in certain instances a full description of the domains of attraction. Our approach will be geometrical. Symmetry will play an important role.

The charm of the limit theory for coordinatewise maxima is its close relationship with multivariate distribution functions. The univariate marginals allow a quick check to see whether a multivariate limit is feasible and what its marginals will look like. Linear and even non-linear monotone transformations of the coordinates are easily accommodated in the theory. Multivariate distribution functions provide a simple characterization of the max-stable limit distributions and of their domains of attraction. Weak convergence to the max-stable distribution function has almost magical consequences. In the case of greatest practical interest, positive vectors with heavy tailed marginal distribution functions, it entails convergence of the normalized sample clouds and their convex hulls.

Distribution functions are absent in our approach. They are so closely linked to coordinatewise maxima that they do not accommodate any other interpretation of extremes. Moreover, distribution functions obscure an issue which is of paramount importance in the analysis of samples, the convergence of the normalized sample cloud to a limiting Poisson point process. Probability measures and their densities on $\mathbb{R}^d$ provide an alternative approach which is fruitful both in developing the theory and in handling applications. The theory presented here may be regarded as a useful complement to the multivariate theory of coordinatewise maxima.

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